

# The Isgur-Wise Function: A Lattice Determination from Pseudoscalar → Pseudoscalar Form Factors

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Form factors for pseudoscalar → pseudoscalar decays of heavy-light mesons are found in quenched lattice QCD with heavy-quark masses in the range of approximately 1-2 GeV. The Isgur-Wise function,  $\xi(\omega)$ , is extracted from these form factors. Results are in good agreement with  $\xi(\omega)$  derived from CLEO measurements for  $B \rightarrow D^* \mu \nu$ .

## 1. THE ISGUR-WISE FUNCTION

Matrix elements are parameterized in terms of two form factors  $h_{\pm}$

$$\frac{\langle B(\vec{p}_b) | V^\mu | A(\vec{p}_a) \rangle}{\sqrt{m_a m_b}} = h_+(\omega; m_a, m_b) (v_a + v_b)^\mu +$$

$$h_-(\omega; m_a, m_b) (v_a - v_b)^\mu \quad (1)$$

where  $v_a$  and  $v_b$  are the meson four-velocities and  $\omega = v_a \cdot v_b$ .

In the heavy quark limit,  $m_{Q_a, b} \rightarrow \infty$ , form factor  $h_-$  tends to zero while  $h_+$  approaches  $\xi(\omega)$ , the universal Isgur-Wise form factor[1].

At finite heavy quark mass,  $h_{\pm}$  are still related to  $\xi(\omega)$  although there are now both short-distance perturbative corrections and nonperturbative corrections in powers of  $1/m_Q$ . Neglecting the power law corrections,

$$h_+(\omega) = \left[ \hat{C}_1 + \frac{\omega+1}{2} (\hat{C}_2 + \hat{C}_3) \right] \xi_{\text{ren}}(\omega) \quad (2)$$

$$h_-(\omega) = \frac{\omega+1}{2} \left[ \hat{C}_2 - \hat{C}_3 \right] \xi_{\text{ren}}(\omega) \quad (3)$$

where the Wilson coefficients  $\hat{C}_i$  have been computed at next-to-leading order by Neuberger[2].

## 2. METHODOLOGY

An  $O(a)$ -improved fermion action[3] was used to generate fermion propagators for 60 quenched gauge configurations on a  $24^3 \times 48$   $\beta = 6.2$  lattice[4]. The three light-quark masses,  $m_q$ , and

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the four heavy-quark masses,  $m_Q$ , used here are also used in our study of  $f_D$  and  $f_B$  on these same configurations[5]. Estimating the heavy-quark mass by the spin-average of the heavy-light pseudoscalar (P) and vector (V) meson masses, in the  $m_q \rightarrow 0$  limit, we find,  $m_Q \approx 1.5, 1.9, 2.1$ , and 2.4 GeV. The light-quark masses in ratio to strange quark mass are  $m_q/m_s \approx 0.41, 0.68$ , and 1.3.

We study euclidean three-point correlation functions

$$G^\mu(0, t, t_b; m_{Q_a}, m_{Q_b}, m_q, \vec{p}_a, \vec{p}_b) = \sum_{\vec{x}, \vec{y}} e^{i\vec{p}_b \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \langle P_b(\vec{x}, t_b) V^\mu(\vec{y}, t) P_a^\dagger(\vec{0}, 0) \rangle \quad (4)$$

where  $\vec{q} = \vec{p}_b - \vec{p}_a$ . Operator  $P_a^\dagger$  creates a  $Q_a \bar{q}$  pseudoscalar and  $P_b$  annihilates a  $Q_b \bar{q}$  pseudoscalar. The current  $V^\mu$  is a local  $O(a)$ -improved vector current[6].

We set  $t_b = 24$  and symmetrize correlators about this time. Correlators have lattice momenta  $\vec{k}_b = (12a/\pi)\vec{p}_b = (0, 0, 0), (1, 0, 0)$ , and  $0 \leq |\vec{k}_a|^2 \leq 2$ . Quark mass  $m_{Q_b}$  can be either 2.4 or 1.9 GeV.

The ratio of a matrix element to the temporal component of the forward matrix element of the flavor-conserving current is extracted by taking the ratio of three-point functions

$$\frac{G^\mu(0, t, t_b; m_{Q_a}, m_{Q_b}, m_q, \vec{p}_a, \vec{p}_b)}{G^4(0, t, t_b; m_{Q_a}, m_{Q_b}, m_q, \vec{p}_b, \vec{p}_a)} \xrightarrow[t_b \gg t \gg 0]{} \frac{Z_a(\vec{p}_a') E_b}{Z_b(\vec{p}_b') E_a} \times \frac{\langle B(\vec{p}_b) | V^\mu | A(\vec{p}_a) \rangle}{\langle B(\vec{p}_b) | V^4 | B(\vec{p}_b) \rangle} e^{-\delta E t}. \quad (5)$$

For all Lorentz components in the ratio that

Table 1  
 $\rho^2$  vs  $\xi(\omega)$  model for parameter set  $\mathcal{A}$ .

BSW	pole	ISGW	linear
0.92 ( $^{+20}_{-18}$ )	0.89 ( $^{+19}_{-17}$ )	0.83 ( $^{+13}_{-13}$ )	0.73 ( $^{+16}_{-14}$ )

are non-zero, a single minimal  $\chi^2$  fit, using the full correlation matrix, is found for the  $t$  dependence in Eqn. 5. Field normalizations  $Z_{a,b}$ , energies  $E_{a,b}$ , and  $\delta E = E_a - E_b$  are constrained to values obtained in fits to the meson propagators[5]. Equation 1 is used with Eqn. 5 to find  $h_{\pm}(\omega; m_a, m_b) / h_+(1; m_b, m_b)$ . After extrapolating  $h_{\pm}$  to the  $m_q \rightarrow 0$  limit, relation Eqn. 2 is used to extract the Isgur-Wise function  $\xi(\omega)$  from  $h_+(\omega)$ .

For flavor-conserving matrix elements,  $h_-$  should be exactly zero. To test this, we allow both  $h_{\pm}$  to be free parameters in the  $\chi^2$  fit. For  $m_Q = 1.5$  GeV and  $m_q \rightarrow 0$  we find  $|h_-| \lesssim 0.1$  which is within  $1\sigma$  of zero. We then constrain  $h_-$  to zero in fits for flavor-conserving matrix elements.

### 3. RESULTS

- **Slope Parameter** The slope parameter,  $\rho^2 \equiv -\xi'(1)$ , is extracted by finding a minimum  $\chi^2$  fit of the lattice  $\xi(\omega)$  to some possible forms for the Isgur-Wise function

$$\xi_{BSW}(\omega) = \frac{2}{\omega + 1} \exp \left( (1 - 2\rho_{BSW}^2) \frac{\omega - 1}{\omega + 1} \right) \quad (6)$$

$$\xi_{pole}(\omega) = \left( \frac{2}{\omega + 1} \right)^{2\rho_{pole}^2} \quad (7)$$

$$\xi_{ISGW}(\omega) = \exp(-\rho_{ISGW}^2(\omega - 1)) \quad (8)$$

as discussed in References [7], [2], and [8] respectively. Values obtained for  $\rho^2$  should be relatively insensitive to the choice of parameterization since Equations 6-8 differ only at  $O((\omega - 1)^2)$ .

In the continuum limit, the forward matrix element of the flavor-conserving vector current has a known normalization. On the lattice, matrix elements are normalized by  $\langle B(\vec{p}_b) | V^4 | B(\vec{p}_b) \rangle$  to reduce lattice artifacts and to cancel the local vector current renormalization  $Z_V$ . It is important to test the consistency of this normalization

Figure 1. The quantity  $V_{cb} \xi(\omega)$  measured by CLEO for  $B \rightarrow D^* \mu \nu$  (diamonds). The lattice form factor (fancy squares) has been scaled by  $|V_{cb}| = 0.034$  as described in the text.

method. We fit lattice form factors to the function  $N \xi_{BSW}(\omega)$  with both  $\rho^2$  and the normalization,  $N$ , determined by the  $\chi^2$  fit. Typically,  $N$  differs from one by  $\lesssim 3\%$  which is within  $1\sigma$ . We then constrained  $N$  to one when finding  $\rho^2$ .

Label as mass set  $\mathcal{A}$  the combination of quark masses:  $m_{Q_b} = 2.4$  GeV,  $m_{Q_a} = \text{any } m_Q$  and  $m_q/m_s \rightarrow 0$ . Values for  $\rho^2$  obtained for this set of masses and Equations 6-8 are shown in Tab. 1. The table also shows  $\rho_{linear}^2$  from  $\xi_{linear} = 1 - \rho_{linear}^2(\omega - 1)$ . Uncertainty estimates are obtained by a bootstrap procedure with only statistical uncertainties shown. The values obtained agree with other determinations[9] and our earlier results[10].

- **Measured Form Factors** In Fig. 1 we compare the lattice form factor for mass set  $\mathcal{A}$  with  $|V_{cb}| \xi(\omega)$  derived from CLEO[11] data for  $B \rightarrow D^* \mu \nu$ . A fit of the CLEO data to  $|V_{cb}| \xi_{BSW}(\omega)$  with  $\rho^2$  constrained to the lattice value  $\rho_{BSW}^2$  of Tab. 1 yields

$$|V_{cb}| = 0.034 \left( ^{+3}_{-3} \right) \left( ^{+2}_{-2} \right) \sqrt{\frac{\tau_B}{1.49 \text{ ps}}} \quad (9)$$

Table 2  
 $\rho_{BSW}^2$  vs  $m_Q$  (GeV).

$m_Q$	1.9	2.4
$\rho^2$	$0.91 (+^{+43}_{-20})$	$1.06 (+^{+66}_{-34})$

The first error is the  $\Delta\chi^2 = 1$  error in the fit to the experimental data and the second error reflects the uncertainty in  $\rho_{BSW}^2$ . Statistical uncertainties in the lattice form factor are of the same size as the errors in the experimental form factor.

The figure shows the lattice  $\xi(\omega)$  from  $P \rightarrow P$  transitions and  $\xi(\omega)$  from CLEO  $P \rightarrow V$  decay data to be remarkably similar in shape. Further studies of heavy quark spin symmetry using  $P \rightarrow V$  three-point functions are underway[12,13].

• **Heavy-Quark Mass Dependence** In Tab. 2 are values for  $\rho_{BSW}^2$  from separate analyses of flavor-conserving matrix elements with  $m_Q = 1.9$  and 2.4 GeV. The errors are large and the change in  $\rho^2$  with  $m_Q$  is only about  $0.5\sigma$  over the range of  $m_Q$  studied.

The  $O(1/m_Q)$  corrections to Eqn. 2 that relates  $h_+(\omega)$  to  $\xi(\omega)$  may be small since, by Luke's theorem[14], there can be at most  $O(1/m_Q^2)$  corrections to this relation at  $\omega = 1$ .

For mass set  $\mathcal{A}$  with  $|\vec{k}_b| = 0$ , the variations in the values of  $\xi(1)$  extracted from  $h_+(1; m_a, m_b)$  are  $\lesssim 0.5\%$  as  $m_{Q_a}$  is varied over the four possible values of  $m_Q$ . The differences are smaller than the statistical uncertainties. For  $|\vec{k}_b| = 1$ , the variations in  $\xi(1)$  values are now as much as ten times larger than for the zero momentum case. However, the differences are still within  $1\sigma$  of zero.

Tests using the relation in Eqn. 3, which is not protected from  $O(1/m_Q)$  corrections by Luke's theorem, are more sensitive indicators of  $m_Q$  effects[15]. A study of  $h_-$  may then help characterize the nonperturbative power law corrections to  $\xi(\omega)$  at finite  $m_Q$ .

• **Light-Quark Mass Dependence** In Tab. 3 we show  $\rho_{BSW}^2$  at fixed light-quark mass for the heavy-quark masses of set  $\mathcal{A}$ . These values should also be compared the value of  $\rho_{BSW}^2$  in Tab. 1 obtained in the chiral limit. The trend is for  $\rho^2$  to decrease with decreasing light-quark mass. Fur-

Table 3  
 $\rho_{BSW}^2$  vs  $m_q/m_s$ .

$m_q/m_s$	0.41	0.68	1.3
$\rho^2$	$1.09 (+^{+24}_{-11})$	$1.19 (+^{+17}_{-10})$	$1.31 (+^{+15}_{-6})$

ther work is necessary to understand the chiral behavior of  $\xi(\omega)$ .

#### 4. CONCLUSION

Using heavy quark symmetry and the lattice is an effective way to study  $B \rightarrow D$  decays.

#### ACKNOWLEDGEMENTS

The authors wish to acknowledge their conversations with C. Bernard, J. Mandula, M. Ogilvie, Y. Shen, and A. Soni concerning this work. This work was carried out on a Meiko i860 Computing Surface supported by SERC Grant GR/32779, the University of Edinburgh and Meiko Limited.

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